A Poisson Analysis of Pit Stops in Formula 1

A Thesis Submitted in Partial Fulfillment of the  
Requirements for the Degree of  
Master of Science

by

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2023

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TABLE OF CONTENTS

Page  
ABSTRACT iii  
INTRODUCTION 1  
METHODS 9  
RESULTS AND DISCUSSION 22  
REFERENCES 35

ii

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An Abstract of the Thesis by  
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This paper attempts to analyze and predict when a pit stop would be made in a Formula1 race. A predictive model could improve efficiency within the race and optimize the race strategy. With lap-by-lap data, the distribution steered us to perform a Poisson analysis. We attempted several Poisson models, including but not limited to, Poisson Regression, Consul’s Poisson, and a Bayesian Poisson Regression. There were issues with over-dispersion which we counteracted with additional models, such as Negative Binomial. None of the models worked. The errors included convergence not being reached for Poisson models as well as maximum tree depth being exceeded in the Bayesian model. It is our recommendation that if further analysis is done on the topic that the researcher should look only at a specific course over a period of time, rather than all courses. It would be ideal to incorporate the weather data which we were unable to do given the data layout.

iii

**Introduction**

Formula 1, also referred to as Formula One and F1, cars are the fastest road-course racing cars in the world – often exceeding 200 mph seconds after braking to below 60 mph. There are ten teams, each with two drivers, competing for two concurrent awards – best driver and best constructor.

The driver with the most points at the end of a season is awarded the Drivers' Championship Award. Teams who have accumulated the most points from their drivers receive the Constructors’ Championship Award. The point breakdown is below:

|  |  |
| --- | --- |
| Position | Points Scored |
| 1 | 25 |
| 2 | 18 |
| 3 | 15 |
| 4 | 12 |
| 5 | 10 |
| 6 | 8 |
| 7 | 6 |
| 8 | 4 |
| 9 | 2 |
| 10 | 1 |
| 11 thru 20 | 0 |

In 2021, Red Bull drivers finished 1st and 4th overall, with the two drivers scoring a combined 585 points. That same season, Mercedes drivers finished 2nd and 3rd overall with the two drivers scoring a combined 613 points. Red Bull had the top driver while Mercedes was deemed the top constructor.

Success in Formula1 predominantly depends on the build of the car. Bell et al. (2016) used a multi-level (random coefficients) linear model to estimate whether driver skill or car construction most effected performance (as reflected by points scored).. They found 86% of the variance in points scored came from the car construction and only 14% of the points came from driver skill.  Kesteren and Bergkamp (2022) used a Bayesian Multilevel Beta regression method to distinguish driver skill from constructor advantage. Their findings also found an 86% variance coming from car construction with 14% of variance from driver skill.

Much of the disparity between cars’ construction can be attributed to budget. Until 2021, there was no cost cap for car design and construction. Asher (2022) breaks down what the cost cap does and does not cover. It does cover all of the parts on the car, transportation costs, most team personnel, garage equipment, etc. Notably, driver salaries as well as the wages of the three highest staff members and travel costs are not included in the salary cap.If a driver can make up 14% of the difference in results, a team with more spending power is at an immediate advantage.

Outside of the car construction capabilities and driver skill, success in F1 racing may be effected by other variables that have not been studied extensively.. The number of races in a season, for example, may effect performance. The past five seasons utilized in this study have ranged from (17) to (22) races per season (a 29% variance), and the current 2023 season will have 23 races..

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Frequency of Courses Per Year** | | | | | | | |
| ***Course #*** | ***Course*** | ***2018*** | ***2019*** | ***2020*** | ***2021*** | ***2022*** | **Sub-Total** |
| *1* | *Abu Dhabi* | 0 | 1 | 1 | 1 | 1 | **4** |
| *2* | *Australia* | 1 | 1 | 0 | 0 | 1 | **3** |
| *3* | *Austria* | 1 | 1 | 2 | 2 | 1 | **7** |
| *4* | *Azerbaijan* | 1 | 1 | 0 | 1 | 1 | **4** |
| *5* | *Bahrain* | 1 | 1 | 2 | 1 | 1 | **6** |
| *6* | *Belgium* | 1 | 1 | 1 | 1 | 1 | **5** |
| *7* | *Brazil* | 1 | 1 | 0 | 1 | 1 | **4** |
| *8* | *Canada* | 1 | 1 | 0 | 0 | 1 | **3** |
| *9* | *China* | 1 | 1 | 0 | 0 | 0 | **2** |
| *10* | *France* | 1 | 1 | 0 | 1 | 1 | **4** |
| *11* | *Germany* | 1 | 1 | 1 | 0 | 0 | **3** |
| *12* | *Great Britain* | 1 | 1 | 2 | 1 | 1 | **6** |
| *13* | *Hungary* | 1 | 1 | 1 | 1 | 1 | **5** |
| *14* | *Italy* | 1 | 1 | 3 | 2 | 2 | **9** |
| *15* | *Japan* | 1 | 1 | 0 | 0 | 1 | **3** |
| *16* | *Mexico* | 1 | 1 | 0 | 1 | 1 | **4** |
| *17* | *Monaco* | 1 | 1 | 0 | 1 | 1 | **4** |
| *18* | *Netherlands* | 0 | 0 | 0 | 1 | 1 | **2** |
| *19* | *Portugal* | 0 | 0 | 1 | 1 | 0 | **2** |
| *20* | *Qatar* | 0 | 0 | 0 | 1 | 0 | **1** |
| *21* | *Russia* | 1 | 1 | 1 | 1 | 0 | **4** |
| *22* | *Saudi Arabia* | 0 | 0 | 0 | 1 | 1 | **2** |
| *23* | *Singapore* | 1 | 1 | 0 | 0 | 1 | **3** |
| *24* | *Spain* | 1 | 1 | 1 | 1 | 1 | **5** |
| *25* | *Turkey* | 0 | 0 | 1 | 1 | 0 | **2** |
| *26* | *United Arab Emirates* | 1 | 0 | 0 | 0 | 0 | **1** |
| *27* | *United States* | 1 | 1 | 0 | 1 | 2 | **5** |
|  | **Sub-Total** | **21** | **21** | **17** | **22** | **22** | **103** |

Furthermore, the courses themselves may impact the car and driver performance Not only does each course follow a unique pattern, but there are also two different road surfaces – track or street. A track course is a dedicated racing track. A street circuit is a collection of public roads that are closed off for the purposes of the F1 race. In 2022, five of the twenty-two (22.7%) courses were street courses. In 2023, 30.4% (seven of the 23) of the races will be on the street.

Finally, climate plays a role in racing conditions and overcall performance. The wide geographic variance of the races and time of year for each race creates significant challenges for both the vehicles and the drivers.. Track temperatures between the coldest and warmest race locations may vary by nearly 100 degrees Fahrenheit . Similarly, humidity varies between arid deserts with practically no humidity to and wetter locales with 92.5% more moisture, . Optimally, any F1 performance analysis would include these factors as they play a significant role in tire degradation and general pit stop strategy. Due to how the weather data is accumulated, it is unable to be merged with the other datasets we have; therefore, we acknowledge access to the data and its importance, but also the inability to properly assess its impact to our study.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Air Temperature (Fahrenheit)** | **Track Temperature (Fahrenheit)** | **Humidity (%)** | **Air Pressure (millibar)** |
| **Low** | 48 | 56.8 | 5 | 779.5 |
| **High** | 99 | 152.6 | 97.5 | 1023.5 |
| **Average** | 74.5 | 95.5 | 54.8 | 986.2 |
| **Standard Deviation** | 41.1 | 49.5 | 18.6 | 49.2 |

This study also does not have access to all of the F1 data which might help the model’s predictive power. Over the course of the race weekend, up to three terabytes of data are generated per the Mercedes-AMG Petronas Formula One Team (2022). This is accumulated through three practice sessions, a qualifying race (which determines starting order), and the final race. This data helps the mechanical engineers adjust the vehicles to increase their performance capabilities.

While, we do not have access to this data, we can obtain some information through the Ergast Developer API and the Formula 1 Official Data Stream. There is a python package called FastF1 used for accessing and analyzing the race results, schedules, timing data, and telemetry. It was created by a German engineer (<https://github.com/theOehrly/Fast-F1>) and much of his work and documentation (<https://theoehrly.github.io/Fast-F1/>) has been used as the foundation of this paper.

The limited nature of historical data available restricts the time period for this study.. Much of the timing data only dates back to 2018 as the information is not publicly available for prior years. As such, this study focused on the years beginning in 2018 and ending in 2022. This five-year period provided 103 races from at least 27 different courses. The uniqueness of courses can be obfuscated by country name as some countries can provide multiple courses such as the United States in 2023 with Miami, Las Vegas, and Austin providing courses. In the analysis, there is not a clear indicator of which track is being used at any given point in time.

. Looking at these races, and acknowledging previous studies found car construction differences account for 86% of the race results, this study sought to examine another factor which might affect the outcome of a race. Once the race starts there is no ability to change the driver or make modifications to the engine. The only possible controllable change once the race starts is the decision to make a pit stop.

Drivers perform pit stops to change tires. Timing of the pit stop and the type of tires utilized on the car both play a significant role in racing. Deciding what tires to put on the car is an optimization problem that has not yet been. This study does not attempt to optimize the tire decision, but rather focuses on whether the need for a pit stop at all may be predicted using a Bayesian Equilibrium analytic model.. The idea stemmed from Bruce Bueno de Mesquita’s book The Predictioneer’s Game (2010) in which he discusses his prediction of political policies individuals will institute.

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We first wanted to look at the distribution of discrete data, comparing laps driven before making a pit stop; similar to how Chimka and Talafuse (2016) used a Poisson Regression to compare golf strokes hit before making it in the hole.

Chart, histogram

Description automatically generated

Chart, histogram

Description automatically generated

Solving for Bayesian Equilibrium requires the use of Python and R to extract, manipulate, and visualize the data. We have global variables, local variables, and random shocks. The choice to utilize Bayesian statistics can be answered using the list provided by Santos et al. (2018) such as the incorporation of prior beliefs, probabilistic estimates, and updating the likelihood lap-after-lap.

Tulabadin and Rudin (2014) performed a similar analysis on NASCAR races. They found that machine learning algorithms such as ridge regression, SVR, LASSO, and random forests are significantly predictive. Their question concerned tire optimization in NASCAR racing, comparing whether it was better to change all four tires or just two. In Formula1, all four tires must be changed at the same time but there is variability as to the type of tire used as there are six variations (Pirelli).

We have already discussed the importance of geography, yet the incapability of incorporating it into our analysis. The personnel involved with the pit stop is similarly difficult to factor into the study.. Each team has twenty mechanics who partake in the pitstop. They can change four tires in two seconds The decision to make the pit stop falls on the “Race Strategy Engineer.” That person uses data, simulation, and feedback to make the decision. The role, however, is not as clearly defined as it seems – some teams do not even identify who their main strategy engineer is and what their role is in the pre-race strategy plans and simulation optimization.



<https://www.caranddriver.com/news/a28567721/fastest-pit-stop-german-grand-prix/>

In addition to determining when a pit stop will be made, the strategist decides on the type of tire to use. Unfortunately even when a pit stop is called for, our data is not 100% accurate. There are six dry tire types, ranging in hardness from C0 (very hard) to C5 (very soft). We only get told if it is soft, medium, or hard. There are also intermediate tires and wet tires. Another consideration with compounds is the requirement that at least two different types of compounds are used in a race.

In addition to the tire compound, the location of competing drivers plays a role in pit stop decision making. It is not uncommon to have a strategy of “do the opposite” of the competing driver. Since pit stops are one of the few things that can be changed during the race and seeing as strategists do care about when another team makes a pit stop, it seems to be a good challenge to try and predict when that pit stop will occur. We are able to show the gap at the end/start of each lap – we cannot ascertain their positioning in between the start and completion of a lap.

Furthermore, an important random event, or shock, that can alter the face of the race is a flag. There are ten distinct flags which have their own meaning. A yellow flag, for example, can be the result of a minor car crash. In a yellow flag all drivers must slow their speed and passing another car is not allowed. As a result, yellow flags are an ideal time to make a pit stop for fresh tiresPretorious (2022) has a chart that shows the breakdown of flags and their meaning:

|  |  |
| --- | --- |
| **Flag Color/Type** | **Meaning** |
| Yellow | Hazard on track. |
| Green | Normal racing conditions apply. |
| Red | Session is suspended. |
| Blue | A faster car is approaching. Move aside (when a driver is being lapped). |
| Yellow & Red Stripes | Track is slippery. |
| Black With an Orange Circle | A driver has a mechanical issue and must return to the pits. |
| Black & White | Warning for unsportsmanlike behavior. |
| Black | Disqualification. |
| White | Slower moving vehicles ahead, or miscellaneous vehicles on the track. |
| Checkered | The session is finished (no new laps may be started). |

This flag data can be found in the “Laps” dataframes under track status. Below is a breakdown of the variables on the Laps dataframe, and underneath that is the breakdown of the various TrackStatus definitions.

|  |  |
| --- | --- |
| **Track Status** | **Description** |
| 1 | Track clear |
| 2 | Yellow flag |
| 3 | Unknown at this time |
| 4 | Safety Car |
| 5 | Red Flag |
| 6 | Virtual Safety Car deployed |
| 7 | Virtual Safety Car ending |

The reason we care about flags is they make a significant impact to the race. They often indicate one or more cars have crashed. In addition to a competitor (or multiple competitors) being out of the running, there is usually a mandatory slow-down of speed between 30% and 40% of normal race-pace. It usually takes teams around 20 seconds to fully complete a pit stop (2-3 seconds for the tire exchange and the rest for driving in/out of the pit stop lane). If every other driver is driving 40% slower, then there are significant time-savings available to drivers utilizing a pit stop while others are still on the course.

**Methods**

Initially the FastF1 package was accessed via JupyterLab. After utilizing JupyterLab via Anaconda for the initial setup, we transferred the workstation to Spyder IDE via Anaconda. Python is where the majority of the programming and analytical work is done. Visualization was done in Python and R. With R, we used Shiny for exploratory data analysis.

Unfortunately, there was no cumulative dataframe for each year. Instead, each race has its own group of dataframes (one for results, one for the lap-by-lap information, one for weather, etc.). This is due to how the data gets collected into the FastF1 package.

Our first step was to put each race into a singular dataframe for the year. The first thing we had to do was to merge the “laps” dataframe with the “results” dataframe for each race. The “laps” dataframe had 27 columns and between 60 and 1533 rows.

The “results” dataframe has 17 columns and 20 rows. In the results dataframe, we had to rename “Abbreviation” into “Driver” to complete the merge as there were no commonly named columns. The results dataframe had multiple duplicate variables as the laps dataframe, so we only grabbed 'Driver', 'GridPosition', 'Position', 'Points'. This helped keep the merged dataframe to a manageable 31 columns. An issue with Results was a programming error that wouldn’t let us view the dataframe in the windowpane like normal. We had to write the Results output to CSV and view it in Excel to understand the data looked like.

After merging the laps and results dataframe, we created a new column called ‘TotalTime’ that calculated the total time a driver had driven. This let us figure out how long it took each driver to reach a certain lap in a race and therefore we were able to sort the race by their total time. The issue we ran across was drivers who did not complete the race, or who were lapped, became intermingled in the final results. The only way to account for those drivers would be to remove them from the dataframe – which would reduce the total number of rows from 110,124 to 3,415, a 97% loss of data.

With our goal of predicting pit stops, we determined that knowing the positioning of each driver at the end of each lap was important. We used the Saudi Arabia race in 2021 as our test data set. We transformed all the timing data into date-time type in seconds and performed column-wise addition to find the total lap time.

The second step was to calculate the pit stop time. The pit stop location on all racecourses is at the start/finish line. This means that whenever a pit stop happens, the “pit stop in” time happens on lap “i” and the “pit stop out” time happens on lap “i + 1”. While this is not intuitive, this is accurate because of the location of the pit stop on the track is right beside the start/finish line on the course. In order to determine the true pit stop time, we used the ‘shift’ function in python to perform the diagonal subtraction of “pit in time” from “pit out time” with a lower bound of 0.

There were quite a number of issues that populated with pit stops. To begin with, everyone’s first lap included a pit stop exit time. This obviously is not true to the race as the drivers do not begin in the pit stop lane. This was overcome with the initial setup of the pit stop calculation by forcing the pit stop time to be 0.

The second issue was the timing associated with the first pit stop. The first record showed a pit out time of 24 minutes 6 seconds and 51 milliseconds. It obviously did not take any driver 24 minutes to be in the pit stop area before the first lap of the race was initiated.

One way to combat the timing was to look at sector times. There are three sectors in a race, and if we sum all three sectors, we should get the total time of the lap per driver. Sector time of a lap per driver is based upon their actual timing in that sector. However, whenever there was a pit stop out time, there was no sector 1 time.

A similarly named variable was the sector time per session. This had additional timing issues - for example, sector 2 time in the Brazil 2021 race in the first observation was 29 seconds and 642 milliseconds. However, there is a sector time per session variable showed the sector 2 time of 1 hour 2 minutes 43 seconds and 773 milliseconds. The difference is the “session” which has nothing at all to do with the start of the race.

We then created a cumulative time per lap by driver. We want to know how much time has elapsed for each driver at the end of each lap to determine their positioning. Pandas has a built-in cumulative summing function that was used in conjunction with the *groupby* method.

We sorted that output by time. We are in the position to know the results of the race. Based off the cumulative time, the ending order should result with the point structure: 26, 18, 15, 12, 10, 8, 6, 4, 2, 1, 0, 0. However, our output was well off: 12, 26, 15, 18, 10, 2, 1, 4, 0, 6, 8, 0 (only 3rd and 7th are accurate)

The issue was mainly with the pit stop times. Due to red flags and potentially other unknown features, the length of pit stop times became inaccurate as the race was re-started and drivers in first place spent longer in the pit stop than others.

We then removed pit stops and just sorted by race time, which resulted in closer to accurate results: 18, 26, 12, 15, 10, 8, 6, 4, 1, 2, 0, 0. There are three positions that are mistaken, and they are all off by exactly one spot (1st vs 2nd, 3rd vs 4th, and 8th vs 9th).

Another attempt was made by removing all the rows with red flags. This change resulted in a final sorting of 12, 26, 15, 18, 10, 2, 1, 4, 0, 6, 8, 0 (seven of the 12 positions are incorrect).

The timing data is a such a known issue within the data that there is an entire column dedicated to determining if the start lap time and end lap time are synced up correctly.

Since our other attempts left us worse off than the original calculation, we stuck with the original calculation with the caveat that sometimes non-finishers are included in the final lap’s data. It was not efficient to continue attempting to manipulate the data to obtain the most accurate results. With the original calculation, we would have to determine the number of non-finishing or lapped drivers who were interspersed in the final results and remove them from the dataframe. This was not feasible to calculate as it was a manual adjustment for 103 races, which would require a comparison of 2060 dataframe rows to real-world results. Furthermore, doing so would not actually solve the problem – it would focus on the top finishers, but the point of the study was to predict pit stops for all drivers at any given lap at any given racecourse.

In addition to the dataframes from the laps and results, there were separate dataframes for weather and messages from race control, and the results. Those were also formed in a similar manner. The reason for including the non-timing data was the impact this data has on the decision to make a pit stop or to stay out. Weather affects tire life and going from a torrential downpour to a blazing sun would require a change of tires. Race control messages can be sent out when there is a yellow flag or other issue that requires drivers to reduce their speed by a certain percentage. Under normal conditions, with the drivers driving at full speed, it is normal for a pit stop to have a driver lose 20 seconds. Under yellow flag conditions, that loss is reduced by 20% to 40%, and they get fresher tires which increases their pace compared to older tires, once they have the approval to go full-speed

The race control messages dataframe would be useful for highly specific analysis to see how a specific team/driver reacts to a change in the field or a penalty. The laps dataframe already has a column called “TrackStatus” that incorporates flags, so we did not look at Race Control Messages closely.

The weather dataframe was also, unfortunately, skipped over. The weather data was accumulated on the minute and had a completely different initialized time compared to the laps dataframe. As such it was impossible to merge the data as the measurements were just on different scales. A future project, figuring out how to incorporate them, would definitely be worthy of exploring.

We then created a “gap” column to indicate the lag each driver has. Basically, we wanted to see how far behind or ahead a driver was compared to the next closest driver.

After getting the data into an imperfect but acceptable format, we performed a Poisson regression on the data. We want to predict when a pit stop is being made so we pared the dataframe down into just those laps involving a pit stop. We also removed duplicate and unnecessary column values. This reduced the columns to 16 and rows to 3415. The total time variable was changed into a total seconds rather than a dtype(‘<M8[ns]’) type.

We were left with five string variables. To get them in the proper form, we had to transform them into categories and then codify them. We can finally start with statistical analysis. The first thing we did was aggregate the pit stop only data by laps, calculating the 'mean', 'min', 'max', 'count'. We had to transform the count into a float. We were then able to visualize the number of pit stops both from a total number and from a normalized viewpoint.

We then looked specifically at the tire life when the pit stop was made, using the same parameters in the aggregated calculations for pit stops by lap.

Our next step was to perform statistics. Sci-kit Learning provides a Python framework for calculating the Poisson regression. It led to a value error in our data using Lap Number as the Y-variable and TyreLife as the Predictor-variable due to array dimensionality. We could reshape the array, but we found another package called *statModels* that allows for Poisson Regression. Around half of the data was able to be summarized without further transformation. For FreshTyres, we replaced with dummy variables. The remaining data that provided issues were all strings (Team, Driver, DriverNumber, Compound, and TrackStatus) that were transformed into categories and then codified.

Looking solely at the Saudi Arabia 2021 race data for only the pit stops, the average lap number a pit stop was made was 14.833. The variance was 46.567 (over three times as much). This breaks the Poisson assumption of an equal mean-variance value, or equally dispersed values. To counteract the overdispersion, we started building Consul’s Generalized Poison regression model (https://www.jstor.org/stable/1267389) , know as GP-1. Some additional errors occurred running this regression. For the “Compound”, “TrackStatus”, “Gap”, “Cumulative”, “CumulativeLap”, “CumulativePit”, “PitTime” factors there was an error where the maximum likelihood optimization failed to converge. The factor “PitInTime” had a divide by zero error. “The factor “PitOutTime” had a linear algebra singular matrix error.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Variable** | **coef** | **std err** | **z** | **P>|z|** | **[0.025** | **0.975]** |
| *Stint* | 1.092 | 0.031 | 35.15 | 0 | 1.031 | 1.153 |
| *Stint\_alpha* | 1.242 | 0.216 | 5.747 | 0 | 0.818 | 1.665 |
| *TyreLife* | 0.137 | 0.005 | 25.99 | 0 | 0.126 | 0.147 |
| *TyreLife\_alpha* | 1.987 | 0.281 | 7.069 | 0 | 1.436 | 2.538 |
| *Sector1TimeSeconds* | 0.037 | 0.001 | 28.66 | 0 | 0.035 | 0.04 |
| *Sector1TimeSeconds\_alpha* | 1.64 | 0.271 | 6.06 | 0 | 1.109 | 2.17 |
| *Sector2TimeSeconds* | 0.051 | 0.001 | 34.54 | 0 | 0.048 | 0.053 |
| *Sector2TimeSeconds\_alpha* | 1.123 | 0.21 | 5.339 | 0 | 0.711 | 1.535 |
| *Sector3TimeSeconds* | 0.037 | 0.001 | 30.33 | 0 | 0.035 | 0.04 |
| *Sector3TimeSeconds\_alpha* | 1.445 | 0.255 | 5.665 | 0 | 0.945 | 1.945 |
| *DriverCat* | 0.198 | 0.008 | 25.49 | 0 | 0.183 | 0.214 |
| *DriverCat\_alpha* | 1.895 | 0.291 | 6.504 | 0 | 1.324 | 2.467 |
| *DriverNumberCat* | 0.202 | 0.008 | 24.01 | 0 | 0.185 | 0.218 |
| *DriverNumberCat\_alpha* | 2.032 | 0.312 | 6.513 | 0 | 1.421 | 2.644 |
| *TeamCat* | 0.376 | 0.017 | 21.98 | 0 | 0.343 | 0.41 |
| *TeamCat\_alpha* | 2.214 | 0.334 | 6.636 | 0 | 1.56 | 2.868 |
| *FreshTyreFalse* | 2.777 | 0.073 | 38.12 | 0 | 2.634 | 2.919 |
| *FreshTyreTrue* | 2.61 | 0.079 | 33.01 | 0 | 2.455 | 2.765 |
| *FreshTyre\_alpha* | 0.437 | 0.137 | 3.191 | 0.001 | 0.168 | 0.705 |

Sorting the data by cumulative race time proved troublesome as the pit stop time played a large role. In a red flag situation, for example, the race stops and restarts many minutes later. Every driver has to come into the pit stop lane during this stoppage so a driver more in the lead would sit longer than someone behind him.

Another potential build upon is Famoye's Restricted Generalized Poison regression model, known as GP-2 (https://www.tandfonline.com/doi/abs/10.1080/03610929308831089). We tested half of the variables, including those which did not have an issue with the normal Poisson model or Consul’s model, and all resulted in a convergence error.

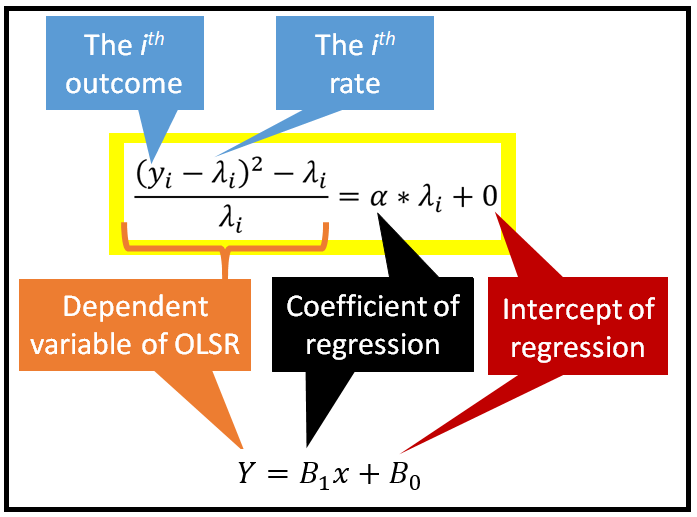
There are only two options left: a Quasi-Poisson process or a negative binomial regression. With the underlying criterion of mean-variance equality being violated, we built a Negative Binomial regression analysis. We could have attempted to fit an additional dispersion parameter using a Quasi-Poisson model, but then we would be unable to directly use model selection or other diagnostic techniques which require an underlying likelihood.

There are two common negative binomial formulas. In Joseph Hilbe’s Negative Binomial Regression book (https://www.cambridge.org/core/books/negative-binomial-regression/12D6281A46B9A980DC6021080C9419E7), he  states that "NB2 is the standard form of negative binomial used to estimate data that are Poisson-over dispersed and is the form of the model which most statisticians understand by negative binomial. NB2 is typically the first model we turn to when we discover that a Poisson model is over dispersed."

The stat models package has an entire method for the NB2 model:

*class statsmodels.genmod.families.family.NegativeBinomial(link=None, alpha=1.0)*

The default value of alpha is 1 which is not necessarily correct. There is a way to estimate the alpha. In Cameron and Trivedi’s book, they propose calculating alpha by using a technique they call auxiliary OLS regression without a constant (https://faculty.econ.ucdavis.edu/faculty/cameron/racd2/)



*“We processed the negative binomial algorithm using a training and test set. Set up the X and y matrices for the training and testing data sets#Add the λ vector as a new column called 'BB\_LAMBDA' to the dataframe of the training data set#add a derived column called 'AUX\_OLS\_DEP' to the pandas DataFrame. This new column will store the values of the dependent variable of the OLS regression#use patsy to form the model specification for the OLSR#Configure and fit the OLSR model.”*

The result for the alpha was a -0.06. The actual alpha should be between 0 and 1. We did take the absolute value in and plugged it into a t-test online calculator to determine the significance. It was significant. It also led to a decent predictive result. It cannot be trusted due to the manual adjustment of the alpha value though:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Variable** | **coef** | **std err** | **z** | **P>|z|** | **[0.025** | **0.975]** |
| *Intercept* | 4.9346 | 26.488 | 0.186 | 0.852 | -46.982 | 56.851 |
| *DriverNumberCat* | 0.0012 | 0.016 | 0.074 | 0.941 | -0.03 | 0.032 |
| *CompoundCat* | -0.0036 | 0.214 | -0.02 | 0.987 | -0.423 | 0.416 |
| *TyreLife* | 0.0001 | 0.033 | 0.004 | 0.996 | -0.064 | 0.065 |
| *FreshTyreCat* | -0.0065 | 0.258 | -0.03 | 0.98 | -0.512 | 0.499 |
| *Stint* | -0.0529 | 0.58 | -0.09 | 0.927 | -1.19 | 1.085 |
| *Gap* | -4.25E-05 | 0.005 | -0.01 | 0.993 | -0.01 | 0.01 |
| *TeamCat* | 0.0005 | 0.045 | 0.011 | 0.991 | -0.087 | 0.088 |
| *DriverCat* | 0.0005 | 0.021 | 0.025 | 0.98 | -0.041 | 0.042 |
| *TrackStatusCat* | 0.0262 | 0.11 | 0.238 | 0.812 | -0.19 | 0.243 |
| *StartPosition* | -0.0038 | 0.022 | -0.17 | 0.867 | -0.048 | 0.04 |
| *EndPosition* | -0.0009 | 0.03 | -0.03 | 0.975 | -0.06 | 0.058 |
| *Points* | 0.0003 | 0.024 | 0.013 | 0.989 | -0.046 | 0.047 |
| *PitTime* | -0.001 | 0.007 | -0.15 | 0.884 | -0.015 | 0.013 |
| *PitInTimeSeconds* | -0.0008 | 0.007 | -0.12 | 0.909 | -0.014 | 0.012 |
| *PitOutTimeSeconds* | -2.58E-13 | 1.38E-12 | -0.19 | 0.852 | -2.97E-12 | 2.46E-12 |
| *Sector1TimeSeconds* | -0.0009 | 0.01 | -0.09 | 0.928 | -0.021 | 0.019 |
| *Sector2TimeSeconds* | -0.0024 | 0.023 | -0.11 | 0.917 | -0.047 | 0.043 |
| *Sector3TimeSeconds* | 0.002 | 0.017 | 0.119 | 0.905 | -0.032 | 0.036 |
| *Cumulative* | 0.0007 | 0.005 | 0.148 | 0.882 | -0.009 | 0.01 |
| *CumulativeLap* | 0.0004 | 0.002 | 0.18 | 0.857 | -0.004 | 0.004 |
| *CumulativePit* | 0.0003 | 0.003 | 0.122 | 0.903 | -0.005 | 0.006 |

Since negative binomial did not work due to the negative alpha, we took a brief glance at a Random Forest regression. The accuracy score for the random forest was 80%. There was a zero-division error for the classification report so results such as the F-score were not available.

|  |  |
| --- | --- |
| **Variable** | **Importance** |
| Sector1TimeSeconds' | 9.421%. |
| CumulativeLap' | 8.47%. |
| Sector2TimeSeconds' | 6.959%. |
| Sector3TimeSeconds' | 6.898%. |
| CumulativePit' | 5.435%. |
| TyreLife' | 4.298%. |
| EndPosition' | 2.19%. |
| TrackStatusCat' | 16.64%. |
| Cumulative' | 11.057%. |
| PitTime' | 10.172%. |
| PitInTimeSeconds' | 10.074%. |
| DriverCat' | 1.543%. |
| Gap' | 1.23%. |
| Points' | 1.067%. |
| Stint' | 1.043%. |
| StartPosition' | 0.891%. |
| TeamCat' | 0.881%. |
| DriverNumberCat' | 0.71%. |
| FreshTyreCat' | 0.592%. |
| CompoundCat' | 0.43%. |
| PitOutTimeSeconds' | 0.0%. |

To perform the random forest, we had to remove the RACE category. We also created a train/test set for prediction purposes.

We then re-ran the mean and variance of the lap number. For Saudi Arabia, the variance was 46.567 with a mean of 14.833. For the full dataset the variance was 225.903 with a mean of 26.847. This has even more over-dispersion than the original analysis.

*Chart, bar chart

Description automatically generated*

We then looked at the mean and variance of the tire compounds. In Saudi Arabia, the mean was .479 with a variance of .297, a difference of 60%. Looking at the full dataset, the mean was 2.904 with a variance of 3.01, a difference of -4%. There is still overdispersion although it’s relatively miniscule.

The difference in variance and mean can be explained by the compound type. The average number of laps a tire undergoes before making a pit stop is dependent upon the type of tire.

As with the Saudi Arabia specific analysis, Consul’s Generalized Poison regression model elicited very little feedback.

Famoye’s Poisson was rife with convergence issues so our next step is Negative Binomial. The first step is to run a GLM Poisson again using test/training data. And this time we found two variables that were not statistically significant (Driver Number and Driver).

Since we tested the Poisson Regression, Consul’s Poisson, Famoye’s Poisson, Random Forest, and a Negative Binomial algorithm on a singular dataframe and at least one of the models appears to work, we moved to apply it to the full dataset.

The next step is to add the lambda column to the dataframe of the training step as it is an essential part to calculate the alpha in the equation

 Unfortunately, as in our Saudi Arabia data, the alpha calculated was a negative 0.029378. Alpha values cannot be negative so nothing further could be done with the Negative Binomial.

Hypothetically, if the alpha was a positive 0.029378, we could have created a predicted vs actual analysis with the following graph as output:

*Chart

Description automatically generated*

It’s not valid though, it’s just included to show that predictive work and visualization can be accomplished if the data was amenable to our goals.

My next attempt was working with a Quasi-Poisson process. There is no set feature in the Python framework that allows for a Quasi-Poisson model to be ran. The one model online which used a Quasi-Poisson process essentially backdoored R into the Python script, transforming the dataframe into an R dataframe and then running the R script on it. When this was done to our data, we were able to get it into the R format but “glm” was not recognized as an R command.

At this point, we decided to simply write the dataframe into a CSV and work on it from RStudio. Unfortunately, this meant that data needed replicated manipulation – such as making the Driver or Team name a factor. After cleaning the data, we installed the PSCL package to allow us to run the statistical tests. In R, the normal Poisson AIC was 20872, the Negative Binomial AIC was similar at 20857. All GLMs use the same log-linear mean function (log(µ) = x >β) but make different assumptions about the remaining likelihood.

We then tried some further statistical tests, such as Generalized Estimating Equations through the R Package, GeePack. This did not work due to the model matrix being rank deficient. We then tried the FlexMix package -https://ro.uow.edu.au/cgi/viewcontent.cgi?referer=&httpsredir=1&article=3410&context=commpapers.

Flex mix did not work due to NAN Log Likelihood Error which can appear for numerous reasons.

The NLME package was considered as well but removed from consideration due to the Gaussian distribution requirement (https://cran.r-project.org/web/packages/nlme/nlme.pdf).

Our next, and final attempt is a Bayesian Analysis of Poisson data. The first step was to run a simulation for the normal model. We did this via the Stan package using the arguments provided in the example we followed, including family = gaussian, prior\_intercept, prior\_aux, prior. Prior should either be the default prior or specified with precedent analysis. Prior\_intercept is the prior distribution for the intercept after centering all the predictors - this also has a default value. Prior\_aux is not always applicable but in the Gaussian family it refers to the “sigma” or standard error deviation. For negative binomial model it refers to “reciprocal dispersion”. Poisson models do not have auxiliary parameters.

We started with two chains of 500 iterations. That led to sampling errors where the estimated Bayesian Fraction of Missing Information was low. With the errors, we updated the chains to 4 and iterations to 1000. Even with four chains, the estimated Bayesian Fraction of Missing Information was low. Bulk Effective Samples Size (ESS) is too low, indicating posterior means and medians may be unreliable. Tail Effective Samples Size (ESS) is too low, indicating posterior variances and tail quantiles may be unreliable. Running the chains for more iterations may help.

We then increased it to 4 chains and 2\*1000 iterations. This took over 15 hours to run and led to two errors – the maximum tree depth was exceeded, and the chains did not mix which was indicated by the lowest R-hat number of 4.22. The next step is to increase the number of iterations, but hardware and timing limitations prevented that attempt.

We did continue to proceed as if we received a useful, or at the least a working, model. So, our next step was to create a posterior prediction using the normal simulation. We plotted the results as a histogram to compare against the original histogram showing the actual lap number that pit stops were made.

Since we know the model is right skewed we can assume that the output would not accurately depict the probability with the normal distribution histogram output. Our next step is to run the Bayesian prior simulation with the Poisson package instead of a gaussian family model. At this point we would summarize the model to determine the specification of the informative priors.

From this step we could add a few lines of code that visualizes n number of prior plausible models dependent upon a specified factor.

Now we can begin the simulation of the posterior for each state by calling the prior model and setting the prior to false. We can then look at how the posterior simulation performs by looking at the MCMC trace, density, and autocorrelation plots. Another good visualization to run is another posterior predictive check. Ideally the newly created histograms would follow the actual histogram. If it doesn’t, then we would need to re-evaluate the arguments or the model itself.

Assuming the model is somewhat accurate, we can actually analyze the results. We can do that analysis visually or by running a confident interval analysis in TidyVerse for the variables. Further detailed analysis can be done by filtering variables to a specific value, such as Driver, Team, or Race.

The final step is to determine the model’s accuracy. We would do this by simulating the posterior predictions, plotting them, summarizing them, and see what how many standard deviations the pit stop lap number is from the posterior mean prediction.

To truly exhibit data science understanding for the thesis as well as model accuracy, one should implement cross validation.

Our data analysis was doing mostly via Poisson but with the over-dispersion, a Negative Binomial approach should not be discounted. Using the same Stan package, we can implement a Negative Binomial model with the same arguments and parameters (adjusting as necessary for the different model).

With failures in Poisson Regression, Consul’s Generalized Poisson Regression, Famoye’s Generalized Poisson Regression, Quasi-Poisson Process, Negative Binomial Regression, Generalized Estimating Equations, FlexMix modeling, and Bayesian Analysis of Poisson data, it is fair to say that analyzing through a Poisson analysis is not the correct attempt for the question of predicting when a pit stop would be made.

**Results and Discussion**

Here is a snapshot of the dataframe for one race, with the results dataframe merged in, against the total laps dataframe before the results data is merged in.

The total dataframe incorporates the RACE name as well as the Year. The Abu Dhabi dataframe incorporates Grid Position, Position, Points and Total Time. The differences are highlighted below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Abu Dhabi 2019** | |  | **Total** | |
| Index | 0 |  | RACE | Australia |
| Time | 0 days 00:35:15.764000 |  | Index | 0 |
| DriverNumber | 44 |  | Time | 0 days 00:08:44.363000 |
| LapTime | NA |  | DriverNumber | 5 |
| LapNumber | 1 |  | LapTime | NA |
| PitOutTime | 0 days 00:00:04.732000 |  | LapNumber | 1 |
| PitInTime | NA |  | PitOutTime | 0 days 00:00:04.103000 |
| Sector1Time | NA |  | PitInTime | NA |
| Sector2Time | 0 days 00:00:43.430000 |  | Sector1Time | NA |
| Sector3Time | 0 days 00:00:40.714000 |  | Sector2Time | 0 days 00:00:24.142000 |
| Sector1SessionTime | NA |  | Sector3Time | 0 days 00:00:36.352000 |
| Sector2SessionTime | 0 days 00:34:35.110000 |  | Sector1SessionTime | NA |
| Sector3SessionTime | 0 days 00:35:15.912000 |  | Sector2SessionTime | 0 days 00:08:08.653000 |
| SpeedI1 | 279 |  | Sector3SessionTime | 0 days 00:08:44.559000 |
| SpeedI2 | 289 |  | SpeedI1 | 275 |
| SpeedFL | 219 |  | SpeedI2 | 290 |
| SpeedST | 293 |  | SpeedFL | 286 |
| IsPersonalBest | FALSE |  | SpeedST | 241 |
| Compound | MEDIUM |  | IsPersonalBest | FALSE |
| TyreLife | 4 |  | Compound | NA |
| FreshTyre | FALSE |  | TyreLife | NA |
| Stint | 1 |  | FreshTyre | NA |
| LapStartTime | 0 days 00:33:31.194000 |  | Stint | NA |
| Team | Mercedes |  | LapStartTime | 0 days 00:07:07.988000 |
| Driver | HAM |  | Team | Ferrari |
| TrackStatus | 2 |  | Driver | VET |
| IsAccurate | FALSE |  | TrackStatus | 1 |
| LapStartDate | 43800.55106 |  | IsAccurate | FALSE |
| GridPosition | 1 |  | LapStartDate | NA |
| Position | 1 |  | Year | 2018 |
| Points | 26 |  |  |  |
| TotalTime | 0 days 00:01:44.570000 |  |  |  |

In total the Abu Dhabi dataframe has a dimensionality of 1075 x 32 while the laps dataframe has a dimensionality of 110,125 x 30

After working through and merging the data across all the years and between dataframes, we ended up with a final dataframe which has a dimensionality of 3416 x 19.

|  |  |  |  |
| --- | --- | --- | --- |
| Index | 116 | 261 | 315 |
| DriverNumberCat | 32 | 1 | 27 |
| LapNumber | 6 | 14 | 18 |
| CompoundCat | 5 | 6 | 6 |
| TyreLife | 5 | 13 | 20 |
| FreshTyre | TRUE | TRUE | FALSE |
| Stint | 1 | 1 | 1 |
| TeamCat | 13 | 14 | 5 |
| Year | 2018 | 2018 | 2018 |
| RACE | Australia | Australia | Australia |
| DriverCat | 5 | 7 | 23 |
| TrackStatusCat | 1 | 0 | 0 |
| GridPosition | 17 | 20 | 2 |
| Position | 19 | 18 | 3 |
| Points | 0 | 0 | 15 |
| TotalTime | 581.634 | 1328.885 | 1627.51 |
| GapSeconds | 6.932 | 10.382 | 13.192 |
| RaceCat | 1 | 1 | 1 |
| FreshTyreCat | 1 | 1 | 0 |

In this final dataframe, we selected only those laps which had a pit stop. We transformed the categorical variables into factors and Boolean variables into integers. At this point, we had a workable dataframe and began with basic analysis.

In the “pit\_in\_time\_lap\_number” dataframe, we looked at which lap number teams and drivers took a pit stop. In this snippet below, team codified as #5, with the driver codified as 23, took 1 pit stop at lap 18 of the race.

|  |  |
| --- | --- |
| Index | 0 |
| Year | 2018 |
| Race | Australia |
| Team | 5 |
| Driver | 23 |
| Mean | 18 |
| Min | 18 |
| Max | 18 |
| NumberOfStops | 1 |

We then visualized the frequency of pit stops and found that predominantly there are only 1 or 2 pit stops per driver per race. We also looked at the normalized data

While frequency of pit stops is good to look at to get a good understanding of the data, why a pit stop is made is dependent on how the tires are handling. Similar to the lap number dataframe and visualizations above, we created a tire life dataframe and visualizations.

|  |  |
| --- | --- |
| Index | 15 |
| Year | 2018 |
| Race | Australia |
| Team | 13 |
| Driver | 16 |
| TyreLife | [19.0, 7.0] |
| Mean | 23.5 |
| Min | 20 |
| Max | 27 |
| NumberOfStops | 2 |

We hand-picked this row of the dataframe to show that there were 2 stops made, one at lap 20 and one at lap 27. That corresponds with a tire life of 19 laps for the first pit stop and 7 laps for the second pit stop. It also shows the mean, minimum, and maximum values regarding the pit stop lap numbers.

With the dataframe prepared and basic analysis done, it was time to actually perform some Poisson modelling. The results for the normal Poisson regression are here:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Variable** | **Coef** | **Std Erro** | **Z** | **P** | **0.025** | **0.975** |
| *TyreLife* | 0.1357 | 0.002 | 61.849 | 0 | 0.131 | 0.14 |
| *Cumulative* | 0.0006 | 8.62E-06 | 74.35 | 0 | 0.001 | 0.001 |
| *CumulativeLap* | 0.001 | 1.31E-05 | 73.849 | 0 | 0.001 | 0.001 |
| *CumulativePit* | 0.0015 | 2.16E-05 | 68.217 | 0 | 0.001 | 0.002 |
| *Sector1TimeSeconds* | 0.0372 | 0.001 | 65.751 | 0 | 0.036 | 0.038 |
| *Sector2TimeSeconds* | 0.05 | 0.001 | 68.866 | 0 | 0.049 | 0.051 |
| *Sector3TimeSeconds* | 0.0371 | 0.001 | 67.139 | 0 | 0.036 | 0.038 |
| *PitTime* | 0.0025 | 4.29E-05 | 58.611 | 0 | 0.002 | 0.003 |
| *PitOutTimeSeconds* | 0 | 0 | nan | nan | 0 | 0 |
| *PitInTimeSeconds* | 0.0004 | 5.19E-06 | 76.482 | 0 | 0 | 0 |
| *Stint* | 1.0934 | 0.015 | 72.19 | 0 | 1.064 | 1.123 |
| *Gap* | 0.0027 | 0 | 13.765 | 0 | 0.002 | 0.003 |
| *Fresh Tyre - FALSE* | 2.8084 | 0.05 | 56.027 | 0 | 2.71 | 2.907 |
| *Fresh Tyre - True* | 2.5713 | 0.056 | 45.564 | 0 | 2.461 | 2.682 |
| *TeamCat* | 0.3661 | 0.007 | 54.139 | 0 | 0.353 | 0.379 |
| *DriverCat* | 0.1939 | 0.003 | 60.429 | 0 | 0.188 | 0.2 |
| *DriverNumberCat* | 0.1997 | 0.003 | 59.707 | 0 | 0.193 | 0.206 |
| *TrackStatusCat* | 0.5792 | 0.01 | 55.519 | 0 | 0.559 | 0.6 |

Moving on to Consul’s Poisson, the results are here:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Variable** | **coef** | **std err** | **z** | **P>|z|** | **[0.025** | **0.975]** |
| *Stint* | 1.0922 | 0.031 | 35.145 | 0 | 1.031 | 1.153 |
| *Stint\_alpha* | 1.2417 | 0.216 | 5.747 | 0 | 0.818 | 1.665 |
| *TyreLife* | 0.1365 | 0.005 | 25.987 | 0 | 0.126 | 0.147 |
| *TyreLife\_alpha* | 1.987 | 0.281 | 7.069 | 0 | 1.436 | 2.538 |
| *Sector1TimeSeconds* | 0.0374 | 0.001 | 28.658 | 0 | 0.035 | 0.04 |
| *Sector1TimeSeconds\_alpha* | 1.6397 | 0.271 | 6.06 | 0 | 1.109 | 2.17 |
| *Sector2TimeSeconds* | 0.0506 | 0.001 | 34.544 | 0 | 0.048 | 0.053 |
| *Sector2TimeSeconds\_alpha* | 1.123 | 0.21 | 5.339 | 0 | 0.711 | 1.535 |
| *Sector3TimeSeconds* | 0.0374 | 0.001 | 30.331 | 0 | 0.035 | 0.04 |
| *Sector3TimeSeconds\_alpha* | 1.4451 | 0.255 | 5.665 | 0 | 0.945 | 1.945 |
| *DriverCat* | 0.1984 | 0.008 | 25.492 | 0 | 0.183 | 0.214 |
| *DriverCat\_alpha* | 1.8954 | 0.291 | 6.504 | 0 | 1.324 | 2.467 |
| *DriverNumberCat* | 0.2015 | 0.008 | 24.005 | 0 | 0.185 | 0.218 |
| *DriverNumberCat\_alpha* | 2.0324 | 0.312 | 6.513 | 0 | 1.421 | 2.644 |
| *TeamCat* | 0.3762 | 0.017 | 21.984 | 0 | 0.343 | 0.41 |
| *TeamCat\_alpha* | 2.2141 | 0.334 | 6.636 | 0 | 1.56 | 2.868 |
| *FreshTyreFalse* | 2.7766 | 0.073 | 38.12 | 0 | 2.634 | 2.919 |
| *FreshTyreTrue* | 2.6103 | 0.079 | 33.005 | 0 | 2.455 | 2.765 |
| *FreshTyre\_alpha* | 0.4366 | 0.137 | 3.191 | 0.001 | 0.168 | 0.705 |

Famoye’s Poisson did not provide any results, only errors.

The next step was a Negative Binomial. Results here:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Variable** | **coef** | **std err** | **z** | **P>|z|** | **[0.025** | **0.975]** |
| *Intercept* | 4.9346 | 26.488 | 0.186 | 0.852 | -46.982 | 56.851 |
| *DriverNumberCat* | 0.0012 | 0.016 | 0.074 | 0.941 | -0.03 | 0.032 |
| *CompoundCat* | -0.0036 | 0.214 | -0.017 | 0.987 | -0.423 | 0.416 |
| *TyreLife* | 0.0001 | 0.033 | 0.004 | 0.996 | -0.064 | 0.065 |
| *FreshTyreCat* | -0.0065 | 0.258 | -0.025 | 0.98 | -0.512 | 0.499 |
| *Stint* | -0.0529 | 0.58 | -0.091 | 0.927 | -1.19 | 1.085 |
| *Gap* | -4.25E-05 | 0.005 | -0.008 | 0.993 | -0.01 | 0.01 |
| *TeamCat* | 0.0005 | 0.045 | 0.011 | 0.991 | -0.087 | 0.088 |
| *DriverCat* | 0.0005 | 0.021 | 0.025 | 0.98 | -0.041 | 0.042 |
| *TrackStatusCat* | 0.0262 | 0.11 | 0.238 | 0.812 | -0.19 | 0.243 |
| *StartPosition* | -0.0038 | 0.022 | -0.168 | 0.867 | -0.048 | 0.04 |
| *EndPosition* | -0.0009 | 0.03 | -0.031 | 0.975 | -0.06 | 0.058 |
| *Points* | 0.0003 | 0.024 | 0.013 | 0.989 | -0.046 | 0.047 |
| *PitTime* | -0.001 | 0.007 | -0.146 | 0.884 | -0.015 | 0.013 |
| *PitInTimeSeconds* | -0.0008 | 0.007 | -0.115 | 0.909 | -0.014 | 0.012 |
| *PitOutTimeSeconds* | -2.58E-13 | 1.38E-12 | -0.186 | 0.852 | -2.97E-12 | 2.46E-12 |
| *Sector1TimeSeconds* | -0.0009 | 0.01 | -0.09 | 0.928 | -0.021 | 0.019 |
| *Sector2TimeSeconds* | -0.0024 | 0.023 | -0.105 | 0.917 | -0.047 | 0.043 |
| *Sector3TimeSeconds* | 0.002 | 0.017 | 0.119 | 0.905 | -0.032 | 0.036 |
| *Cumulative* | 0.0007 | 0.005 | 0.148 | 0.882 | -0.009 | 0.01 |
| *CumulativeLap* | 0.0004 | 0.002 | 0.18 | 0.857 | -0.004 | 0.004 |
| *CumulativePit* | 0.0003 | 0.003 | 0.122 | 0.903 | -0.005 | 0.006 |

The issue with the negative binomial was a negative alpha value, which is mathematically impossible to implement in further analysis. Nonetheless, we did make the negative alpha value an absolute value to run the predicted vs actual analysis, resulting in this graph

Moving on from the Poisson family analysis, we tried random forest next. The results for the importance of variables in a random forest model are here:

|  |  |
| --- | --- |
| **Variable** | **Importance** |
| Sector1TimeSeconds' | 9.421%. |
| CumulativeLap' | 8.47%. |
| Sector2TimeSeconds' | 6.959%. |
| Sector3TimeSeconds' | 6.898%. |
| CumulativePit' | 5.435%. |
| TyreLife' | 4.298%. |
| EndPosition' | 2.19%. |
| TrackStatusCat' | 16.64%. |
| Cumulative' | 11.057%. |
| PitTime' | 10.172%. |
| PitInTimeSeconds' | 10.074%. |
| DriverCat' | 1.543%. |
| Gap' | 1.23%. |
| Points' | 1.067%. |
| Stint' | 1.043%. |
| StartPosition' | 0.891%. |
| TeamCat' | 0.881%. |
| DriverNumberCat' | 0.71%. |
| FreshTyreCat' | 0.592%. |
| CompoundCat' | 0.43%. |
| PitOutTimeSeconds' | 0.0%. |

The accuracy score of the RF model was 42.6% so that was discarded. We could have attempted an XGBoost and that should be looked at in the future.

From Python, we then switched over to analysis in R. We first ran a regular Poisson regression as we did in Python. It resulted in a dispersion parameter to be 1 with null deviance of 31,025.8 on 3414 degrees of freedom, residual deviance of 3762.5 on 3250 degrees of freedom and 20,872 AIC score, on 5 of Fisher Scoring iterations.

We then ran a Quasi-Poisson analysis in R which showed a dispersion parameter to be 1.10401 with null deviance of 31,025.8 on 3414 degrees of freedom, residual deviance of 3762.5 on 3250 degrees of freedom and a non-applicable AIC score, on 5 of Fisher Scoring iterations.

We then ran a negative binomial analysis which showed a dispersion parameter to be 1, null deviance of 28641.5 on 3414 degrees of freedom and residual deviance of 6452.4 on 3250 degrees of freedom with an AIC of 20857 on 1 iteration of Fisher Scoring. The theta was 295.3 with a   standard error of 68.5. A warning occurred while fitting theta that the alternation limit was reached. The 2x log-likelihood was -20525.33

We could not do a hurdle or zip analysis due to non-zero minimum counts.

The GeeGLM was rank deficient.

FlexMix had a non-applicable log-likelihood.

We then attempted a Bayesian Poisson process. We started with two chains of 500 iterations. That lead to sampling errors where the estimated Bayesian Fraction of Missing Information was low. That being said, it still appeared to show some discrepancy when broken down by race. As a whole though, it didn't work. Even though it didn’t work, we did attempt a 10-fold cross validation resampling method.

There were 4 chains where the estimated Bayesian Fraction of Missing Information was low. Bulk Effective Samples Size (ESS) is too low, indicating posterior means and medians may be unreliable. Tail Effective Samples Size (ESS) is too low, indicating posterior variances and tail quantiles may be unreliable. Running the chains for more iterations may help.

We then increased it to 4 chains and 2\*1000 iterations. This took over 15 hours to run and lead to two errors – the maximum tree depth was exceeded, and the chains did not mix which was indicated by the lowest R-hat number of 4.22. The next step is to increase the number of iterations, but limited time prevented that attempt.

A further step which could be taken is making a Bayesian Negative Binomial regression model.

In this paper, we attempted to predict when a driver or team will make a pit stop. We utilized Poisson and related methods. We included all races over a five-year period. Our analysis did not work at any step of the process. Further analysis should inlucde a boosted Random Forest or look towards non-Poisson analysis. If no headway can be made with non-Poisson models, then reducing the data to only specific years or specific racecourses would be advised. The next step should also include a way to incorporate the weather data into our other dataframes. If none of the three prior steps lead to useful results, perhaps a change in question from predicting when a pit stop would be made to what tires a driver will put on the car would be appropriate.

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Python Packages

import fastf1

import numpy as np

import pandas as pd

import fastf1 as ff1

from fastf1.core import Laps

from fastf1 import utils

from fastf1 import plotting

import matplotlib

import matplotlib.pyplot as plt

from matplotlib.collections import LineCollection

from matplotlib import cm

from timple.timedelta import strftimedelta

import os

R Packages

library(tidyverse)

library(readxl)

library(knitr)

library(bookdown)

library(reticulate)

#for shiny

library(shiny)

library(plotly)

library(readxl)

library(tidyverse)

library(magrittr)

library(jpeg)

library(officer)

#for analysis

library(lubridate)

library(pscl)

library(lmtest)

library(geepack)

library(flexmix)

library(lme4)

library(bpr)

library(bayesplot)

library(tidybayes)

library(broom.mixed)

library(rstanarm)

library(bayesrules)